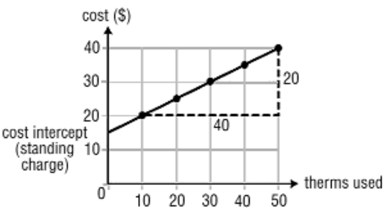
Algebra is used in many areas of mathematics – for example, ARITHMETIC PROGRESSIONS, or number sequences, and Boolean algebra (the latter is used in working out the logic for computers).

In ordinary algebra the same operations are carried on as in arithmetic, but, as the symbols are capable of a more generalized and extended meaning than the figures used in arithmetic, it facilitates calculation where the numerical values are not known, or are inconveniently large or small, or where it is desirable to keep them in an analyzed form.

For example, the following table shows the cost of gas for heating:

therms used	cost ($)
10	20.00
20	25.00
30	30.00
40	35.00
50	40.00



There is a connecting rule between the cost and the number of therms used.

Gradient = change in cost/change in therms:

$$= \frac{40 - 20}{50 - 10} = \frac{20}{40} = \$0.5 \text{ per therm}$$

Cost intercept = S15 (the intercept is the standing charge).

Since this is a straight line GRAPH, a LINEAR EQUATION connecting the cost and therms used can be created:

cost = 0.5 therms + 15 or $c = 0.5\,t + 15$

A straight line graph can be represented by the general formula:

$$y = mx + c$$

where c is the y-intercept, m is the gradient, and (x,y) are the points on the line.

order of calculation The SIMPLIFICATION of an algebraic EQUATION or expression must be completed in a set order. The procedure follows the rules of BODMAS – any elements in **b**rackets should always be calculated first, followed by power **o**f (or index), **d**ivision, **m**ultiplication, **a**ddition, and **s**ubtraction.

For example, to solve the equation:

$$3(2x - x - 1) = 2(x + 3 + 4)$$

collect the like terms and work out the brackets:

$$3(x - 1) = 2(x + 7)$$

multiply out the brackets:

$$3x - 3 = 2x + 14$$

collect the *x*s on the left-hand side of the equation:

$$3x - 3 - 2x = 14$$

then solve for x:

$$x - 3 = 14; x = 14 + 3; x = 17$$

Inequations or INEQUALITIES may be solved using similar rules. When multiplying or dividing by a negative value, however, the direction of the inequality must be reversed, for example: $-x > 5$ is equivalent to $x < -5$.

quadratic equation A QUADRATIC EQUATION is a polynomial equation of second degree (that is, an equation containing as its highest power the square of a variable, such as x^2). The general FORMULA of such equations is:

$$ax^2 + bx + c = 0$$

in which the COEFFICIENTS a, b, and c are real numbers, and only the coefficient a cannot equal 0.

Some quadratic equations can be solved by factorization (see FACTOR (algebra)), or the values of x can be found by using the formula for the general solution.

$$x = \frac{[-b + \sqrt{(b^2 - 4ac)}]}{2a} \quad \text{or}$$

$$x = \frac{[-b - \sqrt{(b^2 - 4ac)}]}{2a}$$

Depending on the value of the discriminant $b^2 - 4ac$, a quadratic equation has two real, two equal, or two complex roots (solutions).

When $b^2 - 4ac > 0$, there are two distinct real roots.

When $b^2 - 4ac = 0$, there are two equal real roots.

When $b^2 - 4ac < 0$, there are two distinct complex roots.

simultaneous equations If there are two or more algebraic equations that contain two or more unknown quantities that may have a unique solution, they can be solved simultaneously as SIMULTANEOUS EQUATIONS. For example, in the case of two linear equations with two unknown variables, such as:

(i) $3y + x = 6$ and

(ii) $3y - 2x = 6$

the solution will be those unique values of x and y that are valid for both equations. Linear simultaneous equations can be solved by using algebraic manipulation to eliminate one of the variables. For example, subtracting equation (ii) from equation (i) gives:

$3y - 3y + x + 2x = 6 - 6$

So $x = 0$, and substituting this value into (ii) gives:

$3y = 6$

So y = 2.

Another method is to rearrange (i) to give:

$x = 6 - 3y$

Substituting this into (ii) gives:

$3y - 2(6 - 3y) = 6$

Multiplying out the brackets gives:

$3y - 12 + 6y = 6$

So $9y = 18$, and $y = 2$.

"Algebra" was originally the name given to the study of equations. In the 9th century, the Arab mathematician Muhammad ibn-Musa al-Khwarizmi used the term *al-jabr* for the process of adding equal quantities to both sides of an equation. When his treatise was later translated into Latin, *al-jabr* became "algebra" and the word was adopted as the name for the whole subject.

The basics of algebra were familiar in ancient Babylonia (*c.* 18th century BC). Numerous tablets giving sets of problems and their answers, evidently classroom exercises, survive from that period. The subject was also considered by mathematicians in ancient Egypt, China, and India. A comprehensive treatise on the subject, entitled *Arithmetica*, was written in the 3rd century AD by Diophantus of Alexandria. In the 9th century, al-Khwarizmi drew on Diophantus' work and on Hindu sources to produce his influential work *Hisab al-jabr wa'l-muqabalah/Calculation by Restoration and Reduction.*

the development of symbolism From ancient times until the Middle Ages, equation-solving depended on expressing everything in words or in geometric terms. It was not until the 16th century that the modern symbolism began to be developed (notably by François Viète) in response to the growing complexity of mathematical statements that were impossibly cumbersome when expressed in words. Farther research in algebra was aided not only because the symbolism was a convenient "shorthand" but also because it revealed the similarities among different problems and pointed the way to the discovery of generally applicable methods and principles.

quarternions and the idempotent law In the mid-19th century, algebra was raised to a completely new level of abstraction. In 1843 Sir William Rowan Hamilton discovered a three-dimensional extension of the number system, which he called "quaternions," in which the commutative law of multiplication is not generally true; that is, $ab \neq ba$ for most quaternions a and b. In 1854 George Boole applied the symbolism of algebra to logic and found it fitted perfectly except that he had to introduce a "special law" that $a^2 = a$ for all a (called the idempotent law).

algebraic structures Discoveries like this led to the realization that there are many possible "algebraic structures," which can be described as one or more operations acting on specified objects and satisfying certain laws. (Thus the number system has the operations of addition and multiplication acting on numbers and obeying the commutative, associative, and distributive laws.)

In modern terminology, an algebraic structure consists of a SET, A, and one or more binary operations (that is, FUNCTIONS mapping $A \times A$ into A) which satisfy prescribed "axioms." A typical example is a structure that had been studied from the 18th century onward and is known as a GROUP. This structure had turned up in the study of the solvability of polynomial equations, but it also appears in numerous other problems (for example, in geometry), and even has applications in modern physics.

modern algebra The objective of modern algebra is to study each possible structure in turn, in order to establish general rules for each structure that can be applied in any situation in which the structure occurs. Numerous structures have been studied, and since 1930 a greater level of generality has been achieved by the study of "universal algebra," which concentrates on properties that are common to all types of algebraic structure.

algebraic fraction in mathematics, fraction in which letters are used to represent numbers – for example, a/b, xy/z^2, and $1/(x + y)$. Like numerical fractions, algebraic fractions may be simplified or factorized. Two equivalent algebraic fractions can be cross-multiplied; for example, if

$$\frac{a}{b} = \frac{c}{d}$$

then $ad = bc$

(In the same way, the two equivalent numerical fractions $2/3$ and $4/6$ can be cross-multiplied to give cross-products that are both 12.)

factorization Algebraic fractions can be simplified by factorization, that is by taking out those factors that are common to both the numerator and denominator. For example, the algebraic fraction:

$$\frac{(x^2 - 2x - 8)(x^2 + 5x + 4)}{(x^2 - 16)(x + 1)}$$

can be simplified as follows:

$(x^2 - 2x - 8)$ factorizes as $(x - 4)(x + 2)$

$(x^2 + 5x + 4)$ factorizes as $(x + 4)(x + 1)$

$(x^2 - 16)$ factorizes as $(x - 4)(x + 4)$

so the $(x - 4)$, $(x + 4)$, and $(x + 1)$ from the denominator cancel out those in the numerator, leaving only $(x + 2)$.

addition and subtraction As with numerical fractions, to add or subtract algebraic fractions a common denominator must be found. The easiest way to do this is to multiply the denominators together. For example:

$$\frac{3}{x} - \frac{4}{x+2} = \frac{3(x+2)}{x(x+2)} - \frac{4x}{x(x+2)}$$

which can be simplified to:

$$\frac{3x + 6 - 4x}{x(x+2)} = \frac{6 - x}{x^2 + 2x}$$

alginate in chemistry, salt of alginic acid, $(C_6H_8O_6)_n$, obtained from brown seaweeds and used in textiles, paper, food products, and pharmaceuticals.

ALGOL contraction of algorithmic language, in computing, an early high-level programming language, developed in the 1950s and 1960s for scientific applications. A general-purpose language, ALGOL is best suited to mathematical work and has an algebraic style. Although no longer in common use, it has greatly influenced more recent languages, such as Ada and Pascal.

algorithm procedure or series of steps that can be used to solve a problem.

In computer science, it describes the logical sequence of operations to be performed by a program. A flow chart is a visual representation of an algorithm.

alias name representing a particular user or group of users in e-mail systems.

This feature, which is not available on all systems, is a matter of convenience as it allows a user to substitute shorter or easier-to-remember real names for e-mail addresses. In 1995 CompuServe announced a system of named aliases for its long, numbered addresses.

alimentary canal tube through which food passes in animals – it extends from the mouth to the anus and forms a large part of the digestive system. In human adults, it is about 9 m/30 ft long, consisting of the mouth cavity, pharynx, esophagus, stomach, and the small and large intestines. It is also known as the gut. It is a complex organ, specifically adapted for DIGESTION and the absorption of food. Enzymes from the wall of the canal and from other associated organs, such as the pancreas, speed up the digestive process.

aliphatic compound any organic chemical compound in which the carbon atoms are joined in straight chains, as in hexane (C_6H_{14}), or in branched chains, as in 2-methylpentane ($CH_3CH(CH_3)CH_2CH_2CH_3$).

Aliphatic compounds have bonding electrons localized within the vicinity of the bonded atoms. CYCLIC COMPOUNDS that do not have delocalized electrons are also aliphatic, as in the alicyclic compound cyclohexane (C_6H_{12}) or the heterocyclic piperidine ($C_5H_{11}N$). Compare AROMATIC COMPOUND.

alizarin or **1,2-dihydroxy-anthraquinone**; $C_6H_4(CO)_2C_6H_2(OH)_2$, derivative from anthraquinone. It is now prepared synthetically from anthracene. Alizarin crystallizes in dark red prisms and sublimes in orange-colored needles, melting at 290°C/554°F. It is almost insoluble in water, but dissolves in alcohol.

It yields with metallic oxides magnificently colored insoluble compounds called "lakes," to which it owes its great value for dyeing purposes. Ferric oxide with alizarin gives a violet-black compound, chromium oxide a claret, calcium oxide a blue, and aluminum and tin give different shades of red.

alk resin obtained from the turpentine tree *Pistacia terebinthus*, which grows chiefly in the Mediterranean region. A yellow to green liquid, Chian or Chio turpentine, is distilled from it.

alkali in chemistry, a BASE that is soluble in water. Alkalis neutralize acids, and solutions of alkalis are soapy to the touch. The strength of an alkali is measured by its hydrogen-ion concentration, indicated by the PH value. They may be divided into strong and weak alkalis: a strong alkali (for example, potassium hydroxide, KOH) ionizes completely when dissolved in water, whereas a weak alkali (for example, ammonium hydroxide, NH_4OH) exists in a partially ionized state in solution. All alkalis have a pH above 7.0.

The hydroxides of metals are alkalis. Those of sodium and potassium are corrosive; both were historically derived from the ashes of plants.

The four main alkalis are sodium hydroxide (caustic soda, NaOH); potassium hydroxide (caustic potash, KOH); calcium hydroxide (slaked lime or limewater, $Ca(OH)_2$); and aqueous ammonia ($NH_{3(aq)}$). Their solutions all contain the hydroxide ion OH^-, which gives them a characteristic set of properties.

alkali metal any of a group of six metallic elements with similar chemical properties: LITHIUM, SODIUM, POTASSIUM, RUBIDIUM, CESIUM, and FRANCIUM. They form a linked group (Group 1) in the PERIODIC TABLE OF THE ELEMENTS. They each have a valence of one and have very low densities (lithium, sodium, and potassium float on water); in general they are reactive, soft, low-melting-point metals. Because of their reactivity they are only found as compounds in nature.

alkaline-earth metal any of a group of six metallic elements with similar bonding properties: beryllium, magnesium, calcium, strontium, barium, and radium. They form a linked group in the PERIODIC TABLE OF THE ELEMENTS. They are strongly basic, bivalent (have a valence of two), and occur in nature only in compounds.

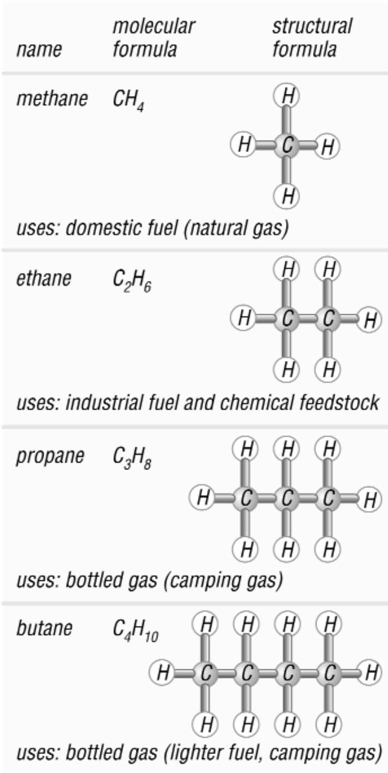
alkaloid any of a number of physiologically active and frequently poisonous substances contained in some plants. They are usually organic bases and contain nitrogen. They form salts with acids and, when soluble, give alkaline solutions.

Substances in this group are included by custom rather than by scientific rules. Examples include morphine, cocaine, quinine, caffeine, strychnine, nicotine, and atropine.

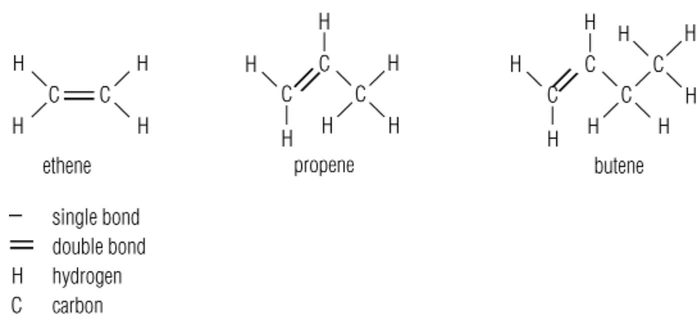
In 1992, epibatidine, a chemical extracted from the skin of an Ecuadorean frog, was identified as a member of an entirely new class of alkaloid. It is an organochlorine compound, which is rarely found in animals, and a powerful painkiller, about 200 times as effective as morphine.

alkane member of a group of HYDROCARBONS having the general formula C_nH_{2n+2}, commonly known as **paraffins**. As they contain only single COVALENT BONDS, alkanes are said to be saturated. Lighter alkanes, such as methane, ethane, propane, and butane, are colorless gases; heavier ones are liquids or solids. In nature they are found in natural gas and petroleum.

alkene member of the group of HYDROCARBONS having the general formula C_nH_{2n}, formerly known as **olefins**. Alkenes are unsaturated compounds, characterized by one or more double bonds between adjacent carbon atoms. Lighter alkenes, such as ETHENE and propene, are gases, obtained from the CRACKING of oil fractions.



alkane The lighter alkanes methane, ethane, propane, and butane, showing the aliphatic chains, where a hydrogen atom bonds to a carbon atom at all available sites.



alkene The alkenes ethene (C_2H_4), propene ($CH_3CH{=}CH_2$), and butene (C_4H_8). Alkenes all have the general formula C_nH_{2n}.